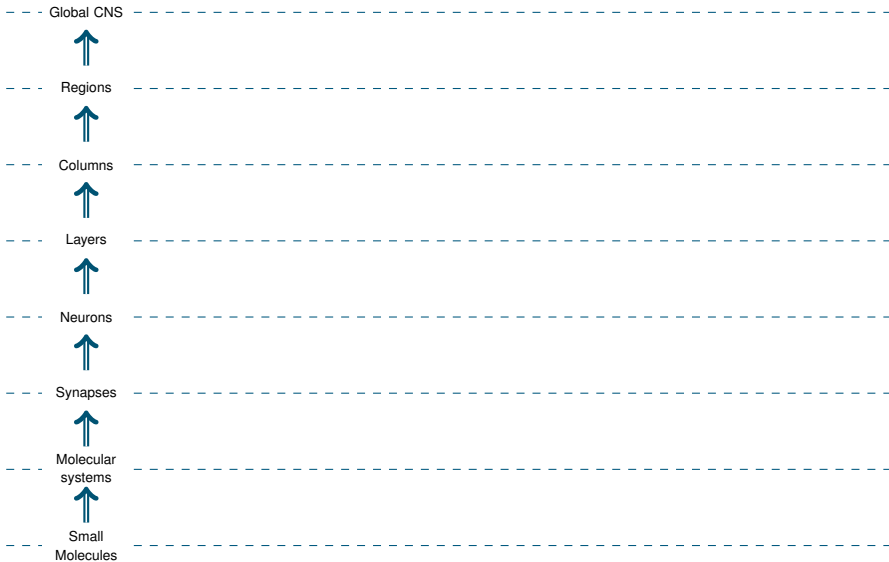
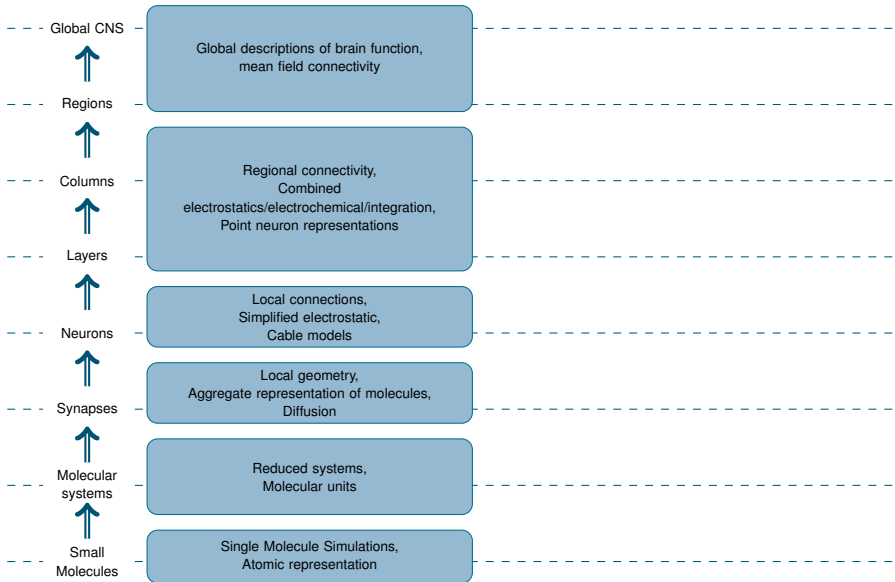


Multiscaling from the Bottom Up

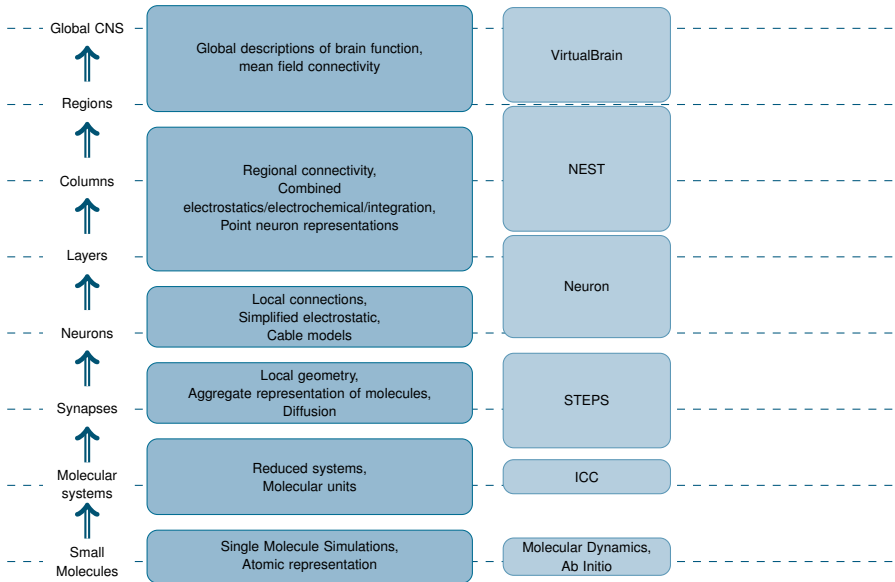
From Molecules to the Nervous System

20 March 2015 | Alexander Peyser | Simulation Lab Neuroscience / Jülich Supercomputing Centre
Institute for Advanced Simulation / Forschungszentrum Jülich

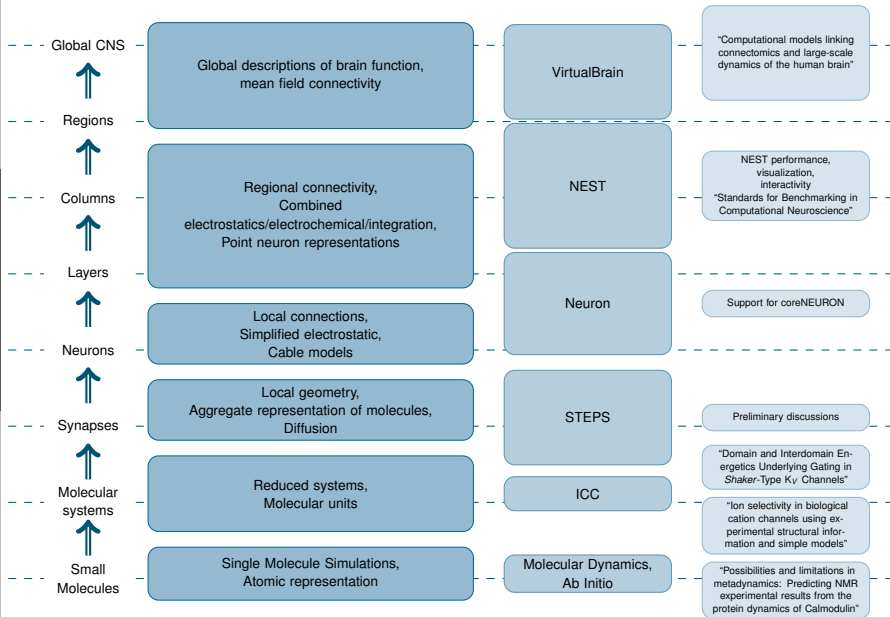




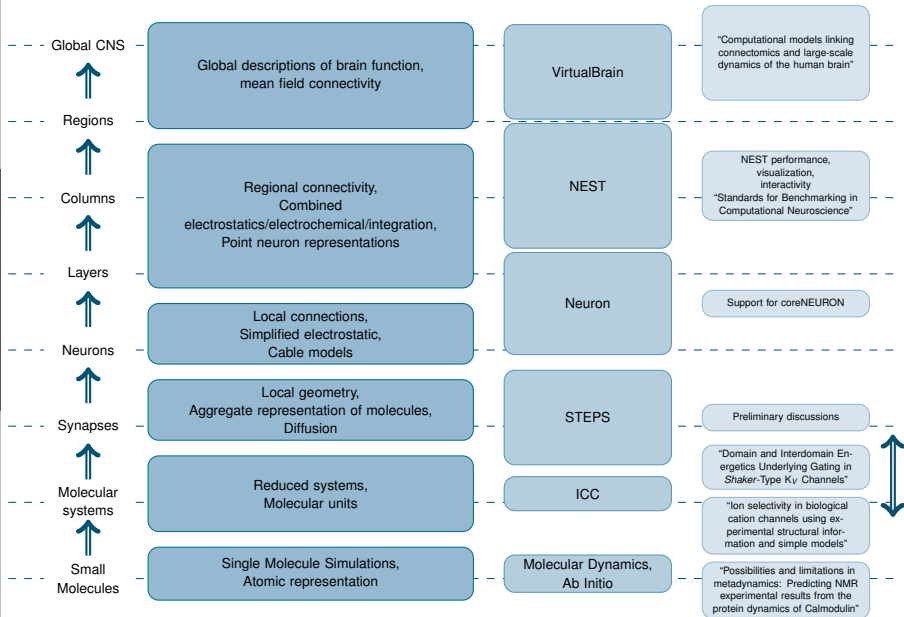
Multiscaling the Nervous System

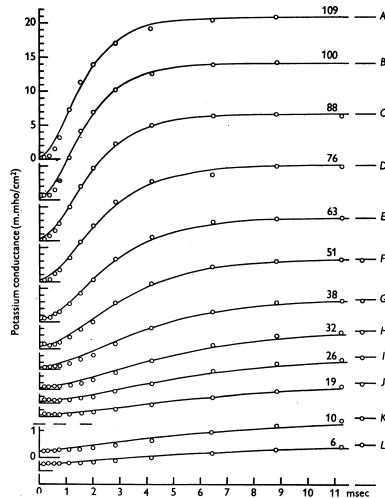


Multiscale the Nervous System



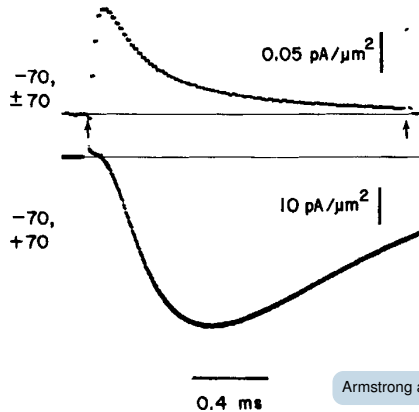
Multiscale the Nervous System





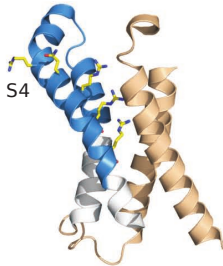
Hodgkin and Huxley (1952, Fig. 3)

$$g_K(t) = g_{K\infty} \left\{ 1 - \left[1 - \sqrt[4]{g_{K0}/g_{K\infty}} \right] \exp(-t/\tau_n) \right\}^4$$

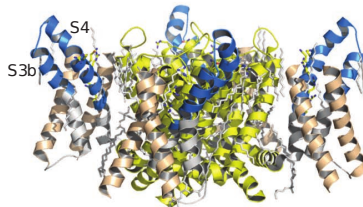
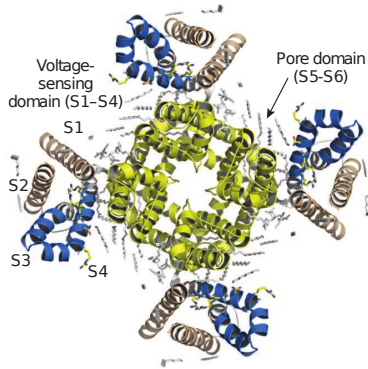


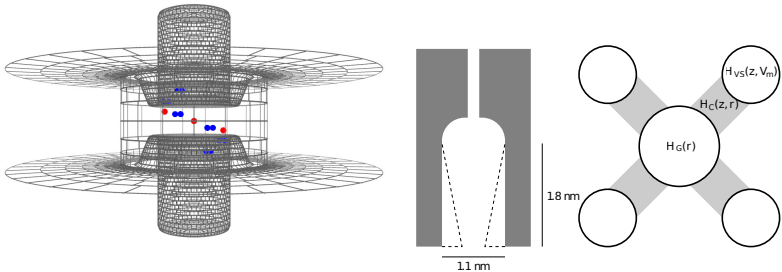
Armstrong and Bezanilla (1973, Fig. 2)

$$g_K(t) = g_{K\infty} \left\{ 1 - \left[1 - \sqrt[4]{g_{K0}/g_{K\infty}} \right] \exp(-t/\tau_n) \right\}^4$$



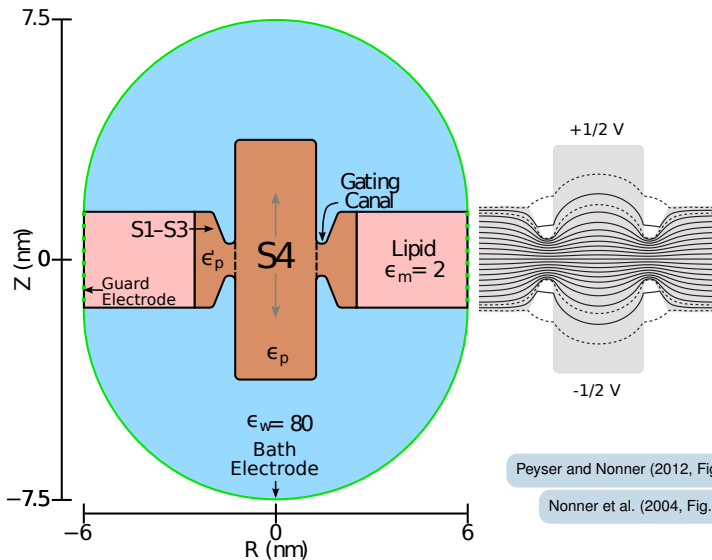
Kalia and Swartz (2013, Fig. 1, 2)





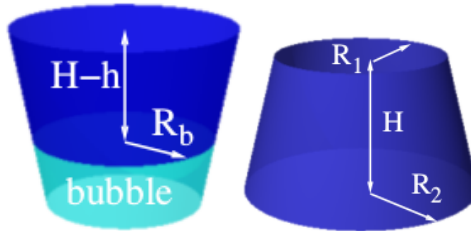
$$B(\mathbf{z}, r, V_m) = \exp\{-\beta[\mathcal{H}_G(r) + \sum_{i=1}^4 (\mathcal{H}_{C,i}(z_i, r) + \mathcal{H}_{B,i}(z_i) + \mathcal{H}_{VS,i}(z_i, V_m))]\}$$

$$\mathcal{H}_{VS,i}(z_i, V_m) = -\beta^{-1} \ln \sum_j \exp[-\beta \mathcal{H}_{VS,i}(z_i, \phi_j, V_m)]$$



Peyser and Nonner (2012, Fig. 1)

Nonner et al. (2004, Fig. 3B)

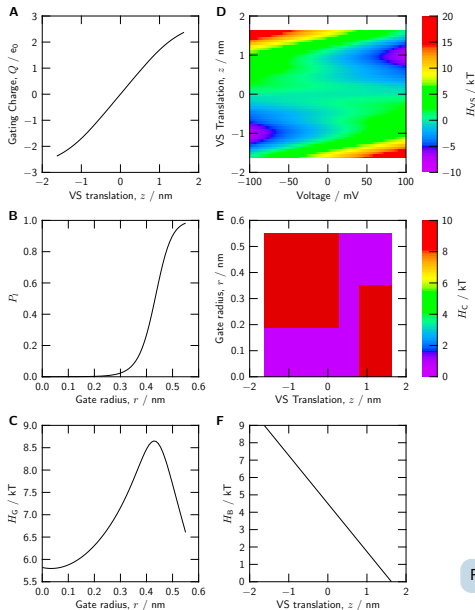


Roth et al. (2008)

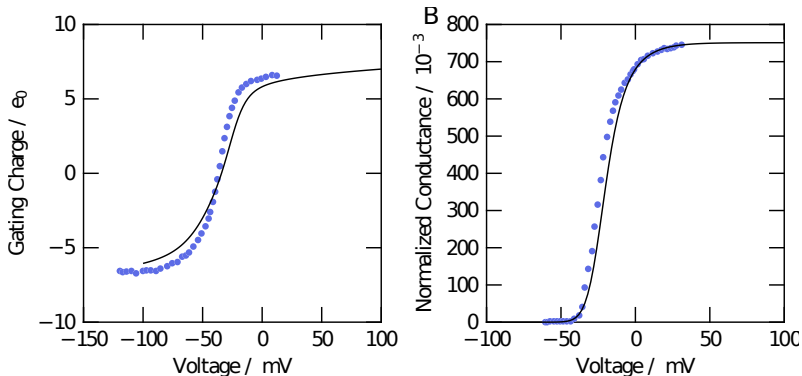
$$P_{\text{open}}(R_2) = (1 + \exp(-\beta \Delta \Omega(R_2)))^{-1}$$

$$\Omega_{\text{gate}}^{\text{open}} = -p_l V(H, R_1, R_2) + \sigma_l M(H, R_1, R_2) + \kappa_l C(H, R_1, R_2)$$

$$\begin{aligned} \Omega_{\text{gate}}^{\text{closed}} = & -p_l V(H - h, R_1, R_b) + \sigma_l M(H - h, R_1, R_b) + \kappa_l C(H - h, R_1, R_b) \\ & - p_g V(h, R_b, R_2) + \sigma_g M(h, R_b, R_2) + \kappa_g C(H - h, R_b, R_2) \\ & + \sigma_{gl} (A(R_b) + A(R_2)) \end{aligned}$$



Peyser et al. (2014, Fig. 2)



Peyser et al. (2014, Fig. 3, black lines)

Seoh et al. (1996, Fig. 2, blue points)

$$Q(V_m) = \sum_{l=1}^{N_r} \sum_{k_1=1}^{N_z} \sum_{k_2=k_1}^{N_z} \sum_{k_3=k_2}^{N_z} \sum_{k_4=k_3}^{N_z} n(k_1, k_2, k_3, k_4) B(z_{k_1}, z_{k_2}, z_{k_3}, z_{k_4}, r_l, V_m)$$

Subcellular simulations: the problem

NEST can handle complex networks

NEURON can represent generic neuronal geometry

Gap

molecular assemblages,
organelles,
subcellular structures
mass diffusion

Atomistic simulations can not be used to build up
macro-molecular structures and organelles

Subcellular simulations: some solutions

- Dryga et al. (2012) Realistic simulation of the activation of voltage-gated ion channels. *Proc Natl Acad Sci U S A*, 109(9):3335–3340, 2012.
- Lopreore et al. (2008) Computational modeling of three-dimensional electrodiffusion in biological systems: Application to the node of Ranvier. *Biophys J*, 95(6):2624–2635, 2008.
- Xylouris et al. (2010) A three-dimensional mathematical model of active signal processing in axons. *Comput Vis Sci*, 13(8):409–418, 2010.
- Hepburn et al. (2012) STEPS: efficient simulation of stochastic reaction-diffusion models in realistic morphologies. *BMC Syst Biol*, 6(1):36, 2012.
- Pods et al. (2013) Electrodiffusion models of neurons and extracellular space using the Poisson-Nernst-Planck equations: numerical simulation of the intra- and extracellular potential for an axon model. *Biophys J*, 105(1):242–254, 2013.

HPC in Neuroscience/SimLab

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Bob Eisenberg
Dirk Gillespie

Universität Tübingen

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Multiscaling the Human Brain

July 21, 2014 | Alexander Peyser | Simulation Lab Neuroscience, Forschungszentrum Jülich

A multiscale model

is composed of hierarchical levels of models, where “upper levels” invoke constraints and closures to combine “lower levels” into computable systems with high fidelity.

The constraints and closures are physical laws and model constraints.

The goal is systems that can be computed within reasonable time and with as good or better fidelity than a primitive model for measures of interest.

- The space to be searched goes as a power of the number of dimensions to be searched: for example $\mathcal{O}(c^n)$ for isotropic flat spaces with a similar size.
- For many spaces, the low energy region becomes a smaller fraction of the total space by the same function

Approximate scales

Simple protein	10^3
Ion channel	10^4
Ribosome	10^5 + associated proteins
RBC	10^{12}
Neuron	10^{17}
Column	$\approx 10^{20}$
Rodent brain	10^{22}
Human brain	10^{25}

The free energy between state A and B , which may or may not be at equilibrium:

$$\exp(-\Delta F_{\mathcal{T}}/kT) = \overline{\exp(-W_{\mathcal{T}}/kT)}$$

This is true for systems that sample their phase space according to the Boltzmann factor:

$$Pr(\mathcal{T}) = \frac{\exp(-\Delta E_{\mathcal{T}}/kT)}{\sum_i \exp(-\Delta E_i/kT)}$$

If we have a system with many trajectories between states and a large fluctuation in work along them ($\sigma \gg kT$), ΔF will be dominated by statistically unlikely low energy pathways.

Calmodulin: An Intrinsically Disordered Protein

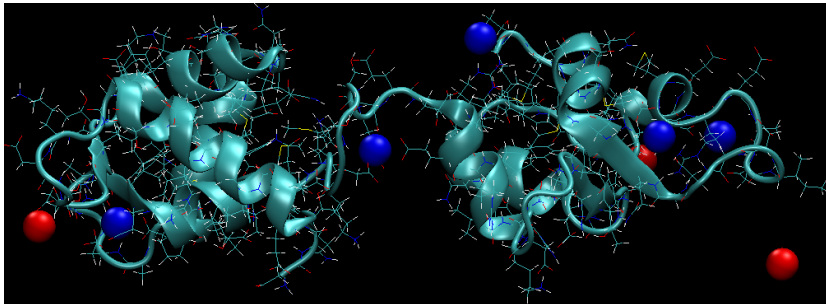
Mass 16706 Daltons

Amino Acids 148

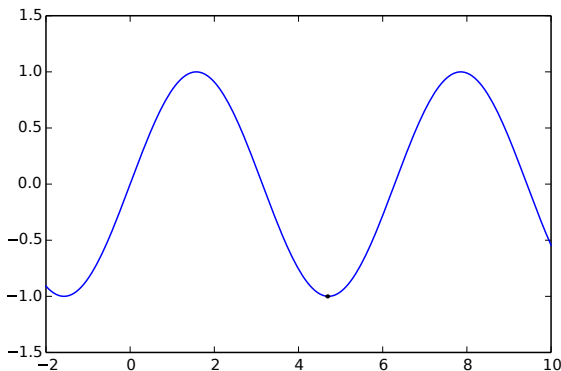
Atoms 1100

Structure Two globular 'heads' connected by a flexible linker
(14 aa)

States binds Ca^{2+} and ubiquitously peptides in a
'compact' state



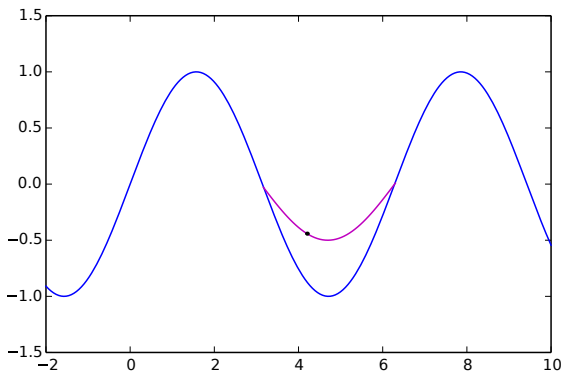
A Metadynamics reduction of molecular dynamics



Underlying energy landscape
Metadynamics force field

• Simulation state

A Metadynamics reduction of molecular dynamics

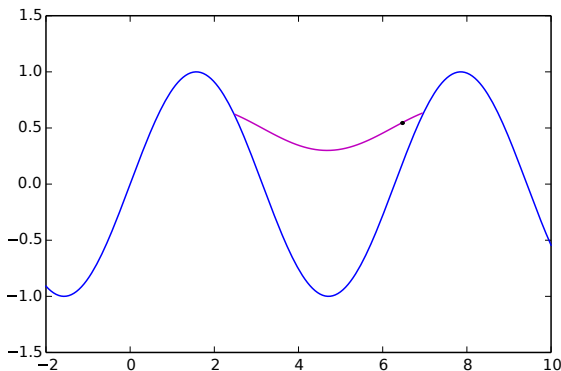


Underlying energy landscape

Metadynamics force field

• Simulation state

A Metadynamics reduction of molecular dynamics

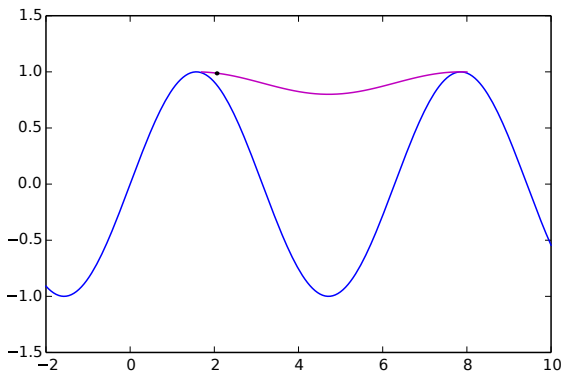


Underlying energy landscape

Metadynamics force field

• Simulation state

A Metadynamics reduction of molecular dynamics

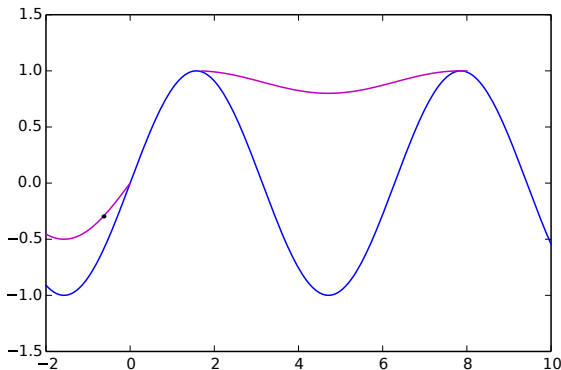


Underlying energy landscape

Metadynamics force field

• Simulation state

A Metadynamics reduction of molecular dynamics



Underlying energy landscape

Metadynamics force field

• Simulation state



Dimensions

Plane Angle Asn60–Glu79–Asp118

Dihedral angle Asn60–Lys74–Glu82–Asp118

Distance Lys75–Glu82

NMR

$$C(t) = 1/5 \langle P_2(\hat{\mu}_{\text{LF}}(0) \cdot \hat{\mu}_{\text{LF}}(t)) \rangle$$

$$J(\omega) = 2 \int_0^\infty (\cos \omega t) C(t) dt$$

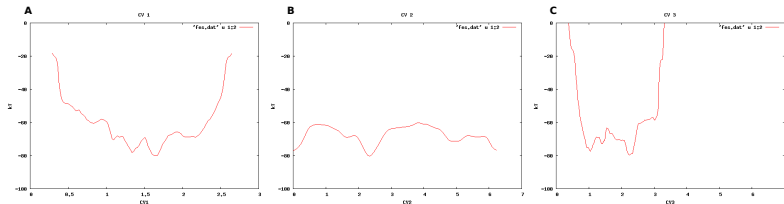
PCS

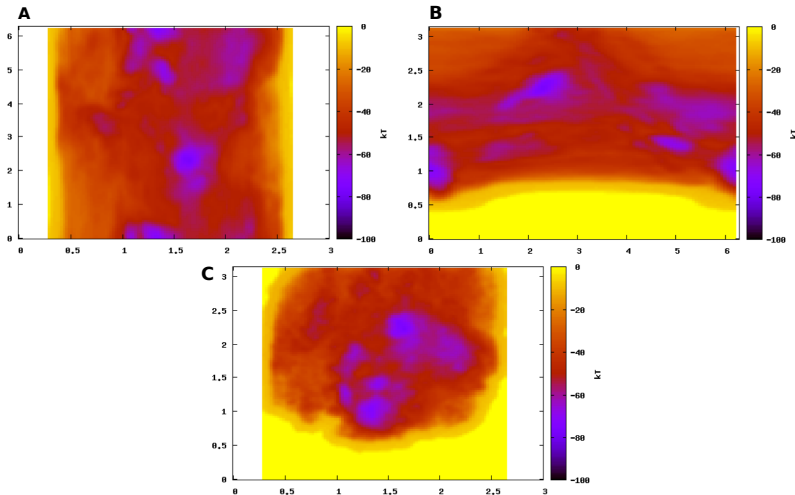
$$\delta^{\text{PCS}} = \frac{1}{12\pi r^3} \left[\Delta\chi_{\text{ax}}(3 \cos^2 \theta - 1) + \frac{3}{2} \Delta\chi_{\text{rh}} \sin^2 \theta \cos 2\varphi \right]$$

RDC

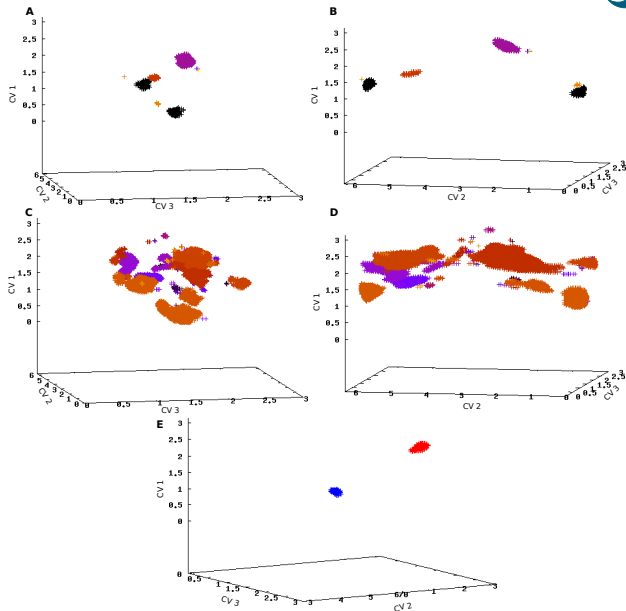
$$\Delta\nu^{\text{RDC}} = - \frac{S_{\text{LS}}}{4\pi} \frac{B_0^2}{15k_B T} \frac{\gamma_A \gamma_B \hbar}{2\pi r_{\text{AB}}^3} \left[\Delta\chi_{\text{ax}}(3 \cos^2 \alpha - 1) + \frac{3}{2} \Delta\chi_{\text{rh}} \sin^2 \alpha \cos 2\beta \right]$$





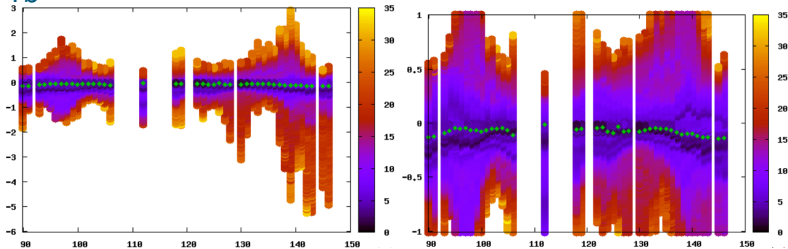


Energy landscapes for axes

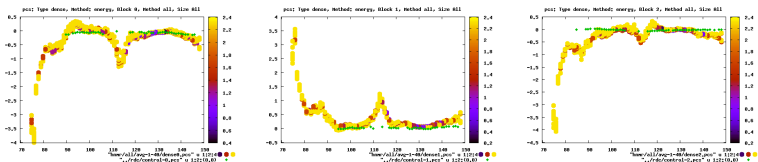
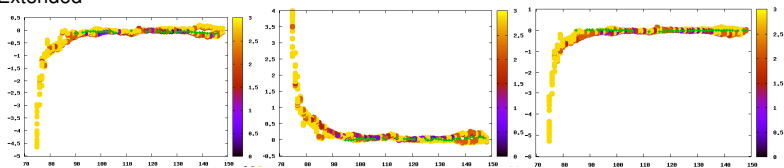


compact
extended

Tb^{3+}

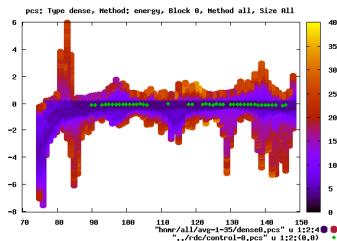


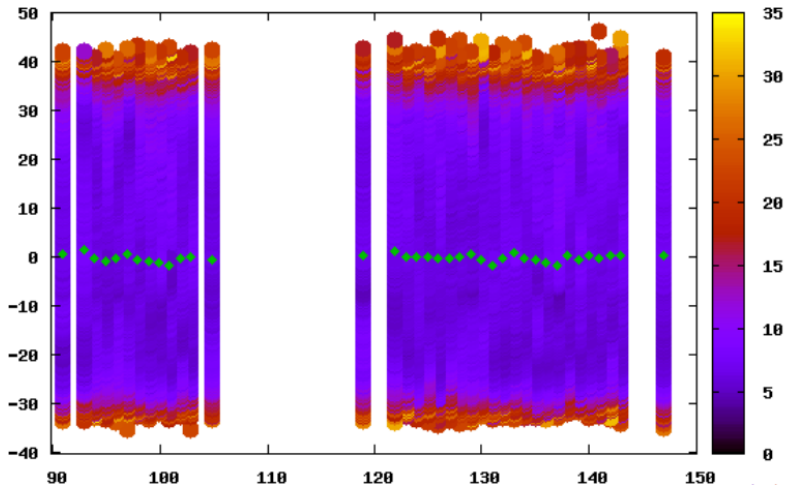
Extended



Compact

Member of the Helmholtz-Association





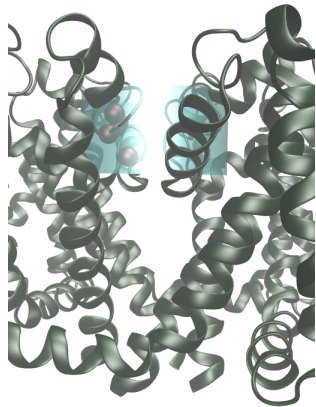
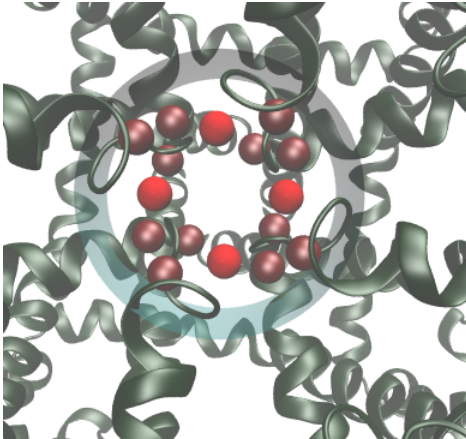
For this regime

- Flat models of non-homogeneous, non-isotropic and distributed systems may give us good values for specific measurables...

But...

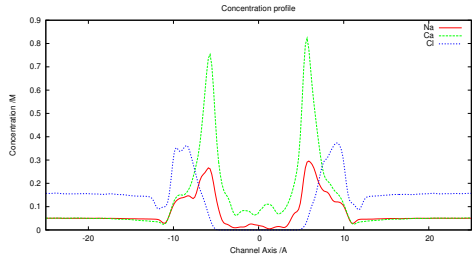
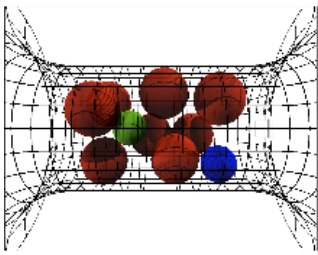
- Computationally expensive
- Weak “explanatory power”
- May have significant artifacts

Coarse Graining: Ca^{2+} vs K^{+} vs Na^{+} selectivity



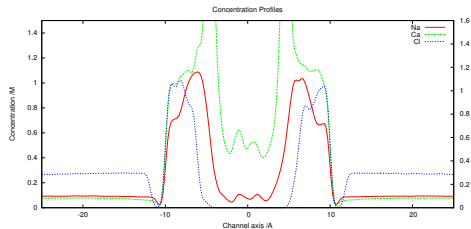
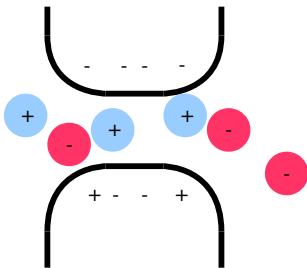
Justin Finnerty

Ca²⁺ channel: Volume model



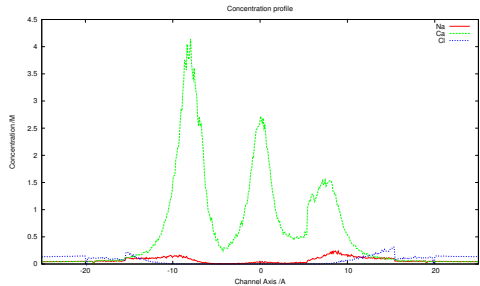
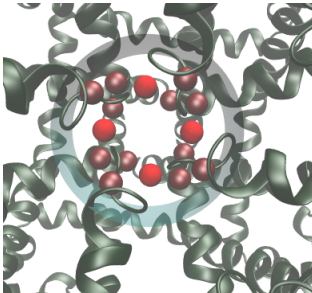
Justin Finnerty

Ca²⁺ channel: Plus dielectrics

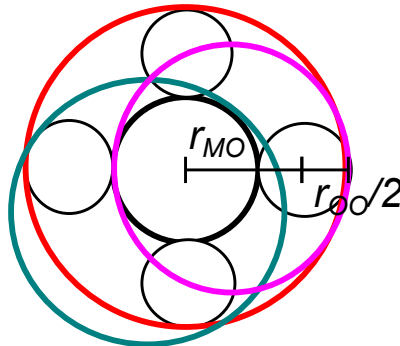


Justin Finnerty

Ca²⁺ channel: Plus localization



Justin Finnerty



	K^+		Na^+		Ca^{2+}	
	$K_{dh(x),K^+}$	$d_{K^+,x}$	$K_{dh(x),Na^+}$	$d_{Na^+,x}$	$K_{dh(x),Ca^{2+}}$	$d_{Ca^{2+},x}$
$x = 6$	1	8.32	1	7.44	1	7.44
5	2.2×10^{-2}	≈ 8.32	5.8×10^{-3}	≈ 7.44	1.8×10^{-16}	≈ 7.44
4	5.5×10^{-5}	6.21	1.7×10^{-5}	5.44	4.4×10^{-34}	5.44
3	2.5×10^{-8}	5.54	3.7×10^{-10}	5.10	8.5×10^{-53}	5.10
2	7.9×10^{-13}	2.8	6.3×10^{-17}	2.0		

Multiple partial hydration state model

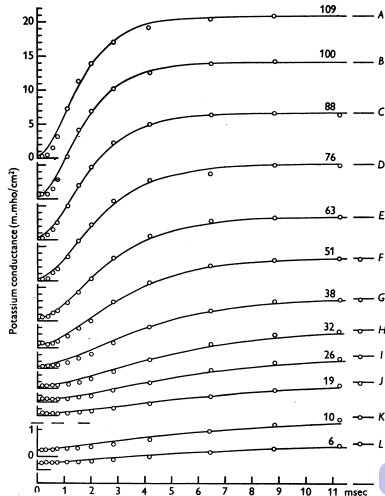
Ions A, B	Hyd.	$\frac{K_{dh(n),A}}{K_{dh(m),B}}$	K_2	α	Expt
KcsA Doyle et al. (1998)					
K^+, Na^+	$n, m \leq 2$	13000	0.1	1,300	> 170 LeMasurier et al. (2001)
NavMs McCusker et al. (2012)					
Na^+, K^+	$n, m \leq 4$	0.3	30	9	11-18 Ulmschneider et al. (2013)
NavAb Payandeh et al. (2011)					
Na^+, K^+	$n, m \leq 4$	0.3	12	2.4	6-30 Finol-Urdaneta et al. (2014)
CavAb Tang et al. (2014)					
Ca^{2+}, Na^+	$(n, m \leq 6)$	1	590	590	380 Tang et al. (2014)

$$\alpha = K_{A,B} \equiv \frac{K_{dh(n),A}}{K_{dh(m),B}} K_2.$$

Reduced models

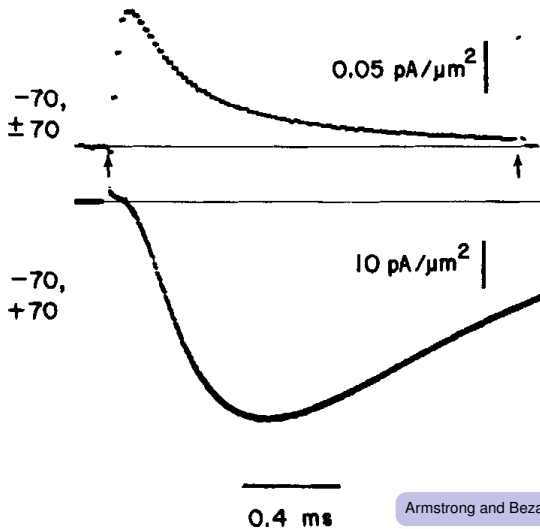
- Allow piecewise development of models
- Have (mathematically) explanatory power
- Naturally fit into multiscale models
- Can be mapped down to atomic scales

Voltage Gated K⁺ Currents

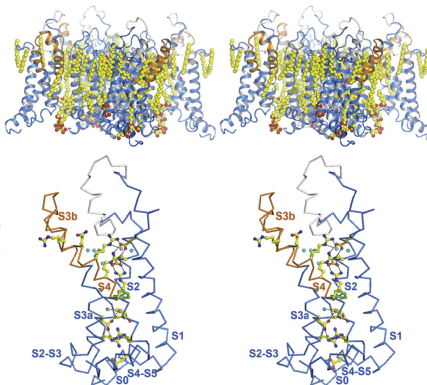
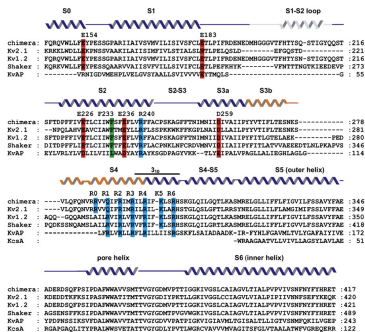


Hodgkin and Huxley (1952, Fig. 3)

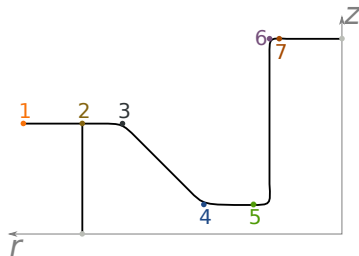
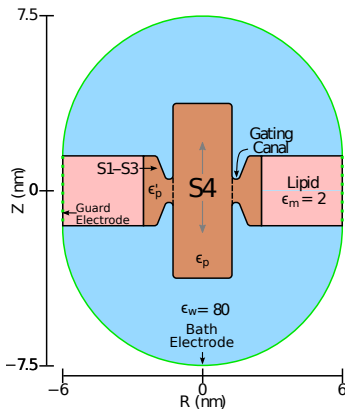
$$g_K(t) = g_{K\infty} \left\{ 1 - \left[1 - \sqrt[4]{g_{K0}/g_{K\infty}} \right] \exp(-t/\tau_n) \right\}^4$$



Armstrong and Bezanilla (1973, Fig. 2)

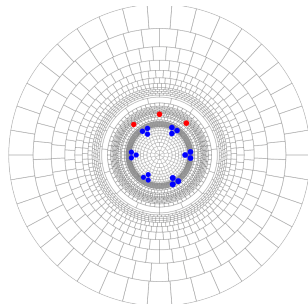
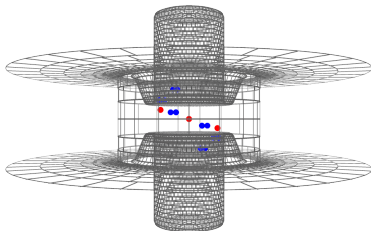


Tao and MacKinnon (2008, Fig. 1)



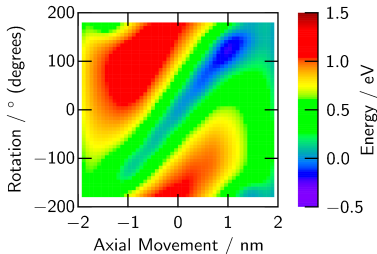
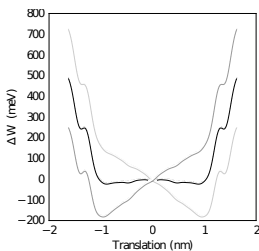
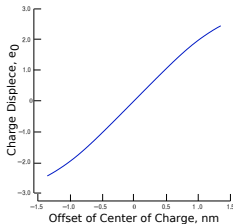
Model		1	2	3	4	5	6	7
α	r	6.0	2.532	1.966	1.566	1.466	1.266	1.0
	z	1.5	1.5	1.5	0.5015	0.5015	3.7515	3.7515
3_{10}	r	6.0	2.492	1.946	1.546	1.446	1.246	0.98
	z	1.5	1.5	1.5	0.602	0.602	4.602	4.602

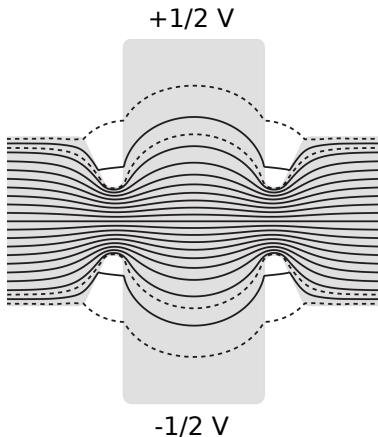
Peyser and Nonner (2012a, Fig. 1)



Peyser and Nonner (2012a, Fig. 2)

Single VS at 1 V: Energy & Displacement





Nonner et al. (2004)

Shockley-Ramo Theorem (Integrated)

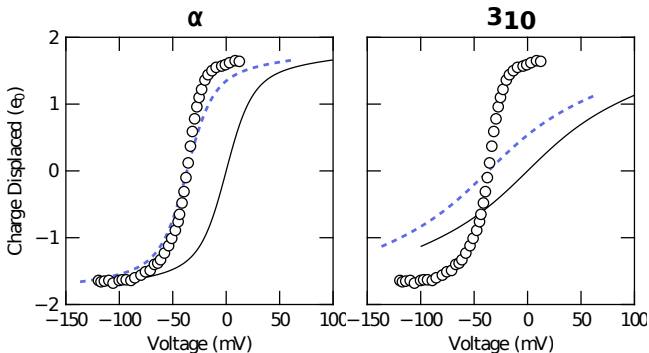
$$Q = - \sum_k q_k V_o(\mathbf{r}_k) / (1V)$$

Shockley-Ramo Energetics

SR can be extended to energetic calculations:

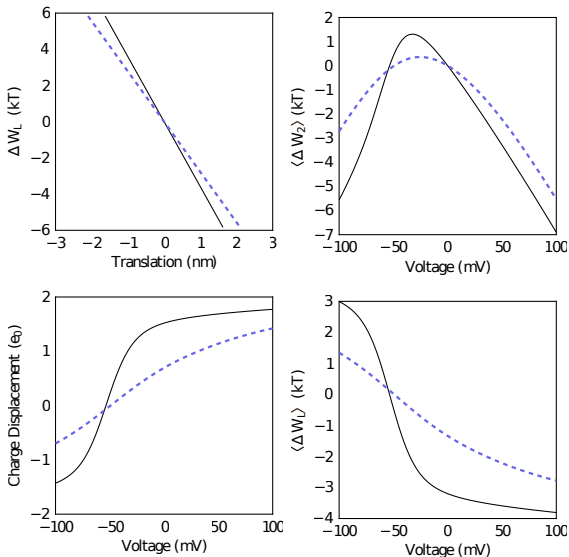
$$W = \frac{1}{2} \sum_k q_k V^{V_E=0}(\mathbf{r}_k) - QV_m$$

All results can be calculated from one SR energy calculation with $V_m = 0$, and one SR displacement calculation $V_m = 1V$ for each conformation.

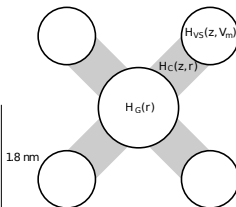
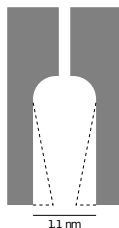
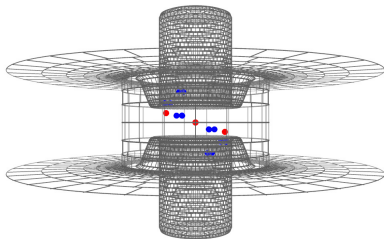


Peyser and Nonner (2012b, Fig. 3)

Seoh et al. (1996, Fig. 2)

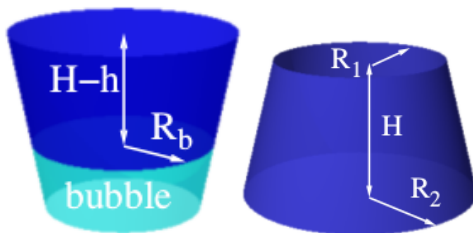


Peyser and Nonner (2012a, Fig. 8)



$$B(\mathbf{z}, r, V_m) = \exp\left\{-\beta[\mathcal{H}_G(r) + \sum_{i=1}^4 (\mathcal{H}_{C,i}(z_i, r) + \mathcal{H}_{B,i}(z_i) + \mathcal{H}_{VS,i}(z_i, V_m))]\right\}$$

$$\mathcal{H}_{VS,i}(z_i, V_m) = -\beta^{-1} \ln \sum_j \exp[-\beta \mathcal{H}_{VS,i}(z_i, \phi_j, V_m)]$$

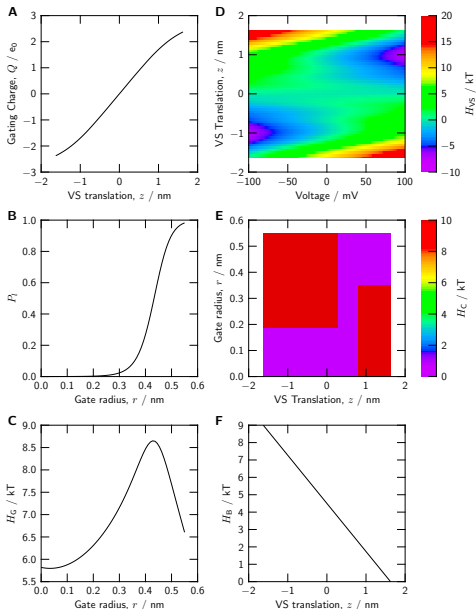


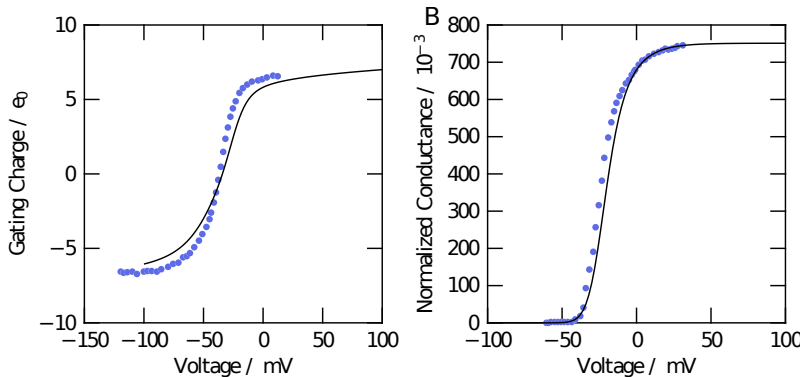
Roland Roth

$$P_{\text{open}}(R_2) = (1 + \exp(-\beta \Delta \Omega(R_2)))^{-1}$$

$$\Omega_{\text{gate}}^{\text{open}} = -p_l V(H, R_1, R_2) + \sigma_l M(H, R_1, R_2) + \kappa_l C(H, R_1, R_2)$$

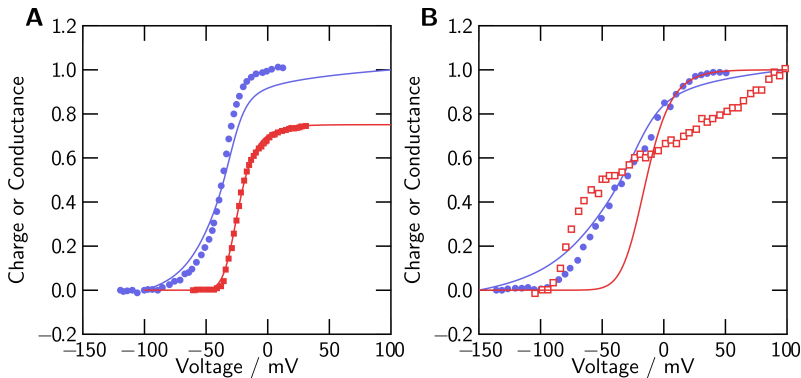
$$\begin{aligned} \Omega_{\text{gate}}^{\text{closed}} = & -p_l V(H-h, R_1, R_b) + \sigma_l M(H-h, R_1, R_b) + \kappa_l C(H-h, R_1, R_b) \\ & -p_g V(h, R_b, R_2) + \sigma_g M(h, R_b, R_2) + \kappa_g C(H-h, R_b, R_2) \\ & + \sigma_{gl}(A(R_b) + A(R_2)) \end{aligned}$$





$$Q(V_m) = \sum_{l=1}^{N_r} \sum_{k_1=1}^{N_z} \sum_{k_2=k_1}^{N_z} \sum_{k_3=k_2}^{N_z} \sum_{k_4=k_3}^{N_z} n(k_1, k_2, k_3, k_4) B(z_{k_1}, z_{k_2}, z_{k_3}, z_{k_4}, r_l, V_m)$$

Mutant of first extracellular VS charge Shaker R362Q



Wolfgang Nonner

Multiscale models

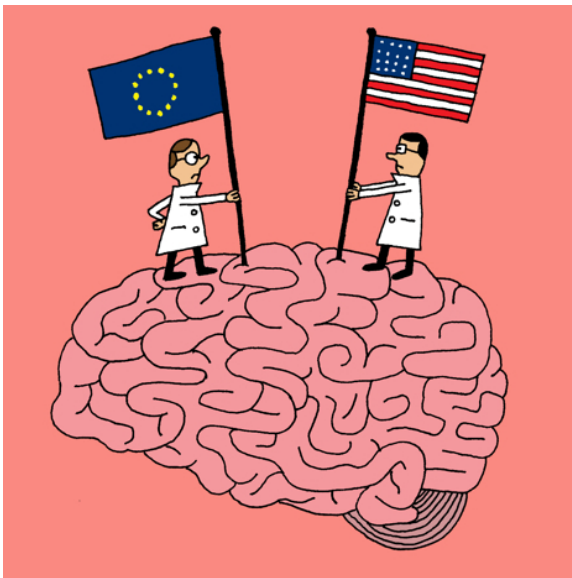
- Allow piecewise development of models
- Have (mathematically) explanatory power
- Decompose systems into useful subsystems
- Look like engineering descriptions
- Can combine disparate physics
- Can naturally be used to formulate falsifiable hypotheses

Overview: Human Brain Project

Boris Orth

HBP

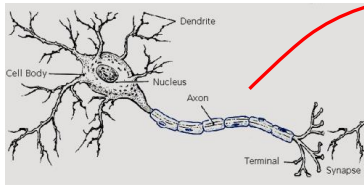
BRAIN
aka
BAM



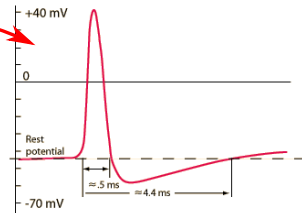
The New Yorker
18 Feb 2013

HBP Nest VisNest

An environment for neural systems simulations



Neuron

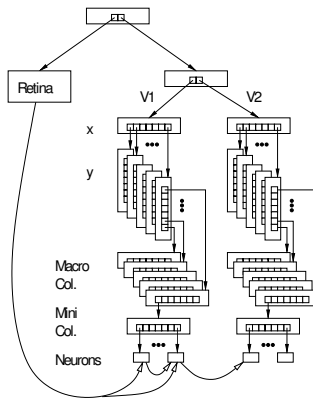
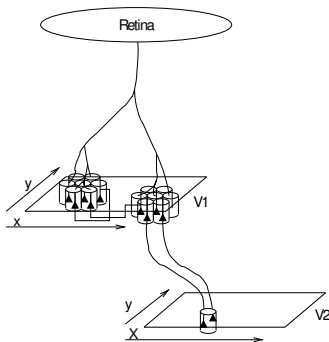


Spike

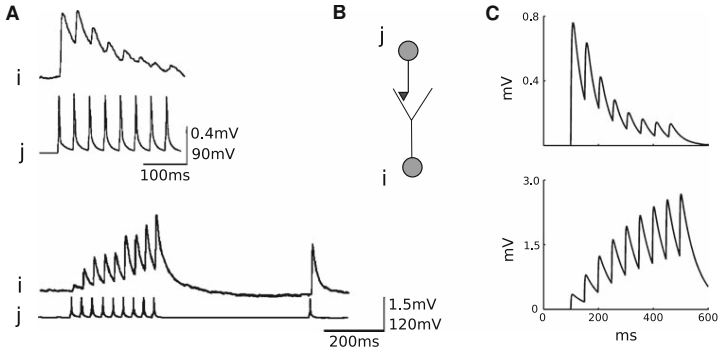
Wolfram Schenk

$$dV/dt = (-V + IR_m)/\tau_m$$

An environment for neural systems simulations



Diesmann and Gewaltig (2001)

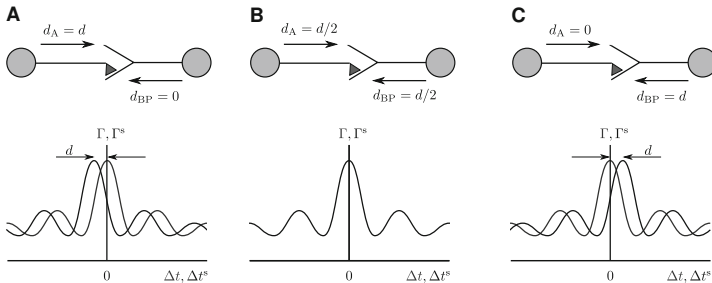


Morrison et al. (2008, Fig. 2)

$$dx/dt = z/\tau_{rec} - u_+x - \delta(t - t_j^f)$$

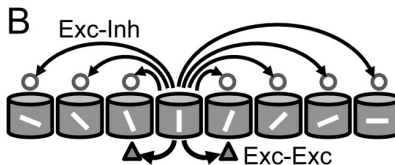
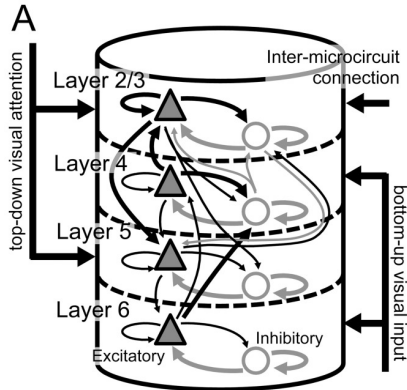
$$dy/dt = -y/\tau_I + u_+x - \delta(t - t_j^f)$$

$$dz/dt = y/\tau_I - z/\tau_{rec}$$



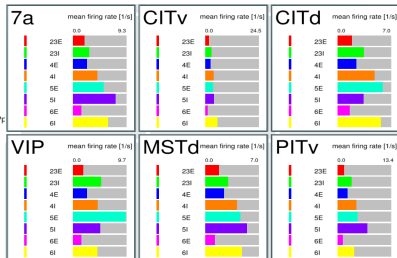
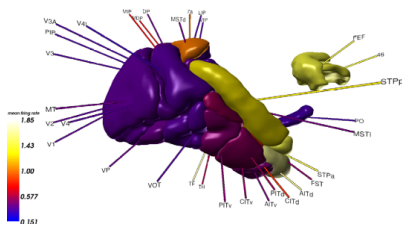
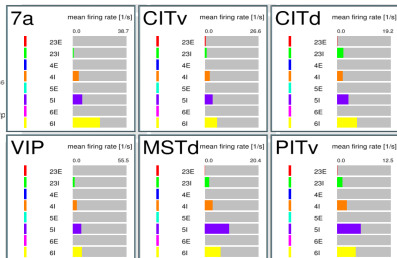
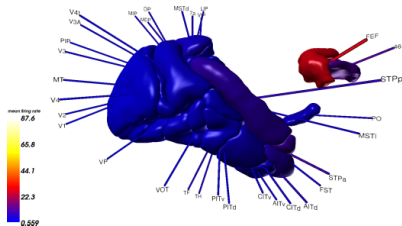
Morrison et al. (2008, Fig. 8)

$$\dot{w} = -F_{-}(w) \int_{-\infty}^0 d\Delta t^s K_{-}(\Delta t^s) \Gamma(\Delta t^s + (d_A - d_{BP})) \\ + F_{+}(w) \int_0^{\infty} d\Delta t^s K_{+}(\Delta t^s) \Gamma(\Delta t^s + (d_A - d_{BP}))$$



Wagatsuma et al. (2013)

Christian Nowke



The gap between molecular models and tissues

From subcellular to supercellular

Neuron is the major project within the HBP modeling neurons at the cable compartment level

Subcellular projects are still in early stages, such as electrically accurate synapse models

Supracellular projects tend to be highly reduced, but are being developed for cardiac and hepatic systems in addition to the brain

Which elements should be studied at each scale and

How to properly link different scales
are questions that are often not answered robustly

This gap is not just a problem of computational tractability, but even more importantly, one of correctness.

HPC in Neuroscience

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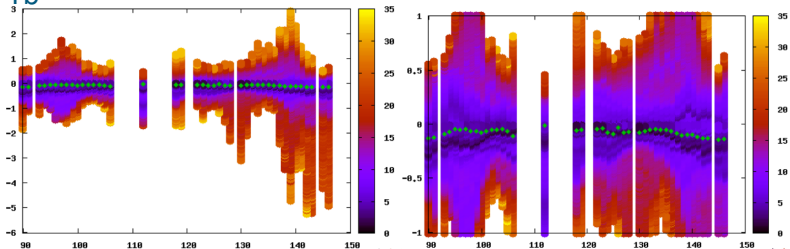
M. B. Ulmschneider, C. Bagnéris, E. C. McCusker, P. G. DeCaen, M. Delling, D. E. Clapham, J. P. Ulmschneider, and B. A. Wallace. Molecular dynamics of ion transport through the open conformation of a bacterial voltage-gated sodium channel. [Proceedings of the National Academy of Sciences](#), 110(16):6364–6369, 2013.

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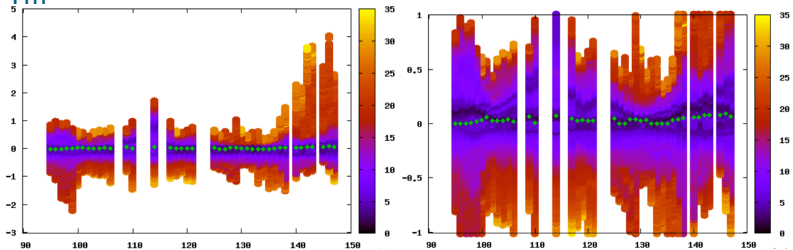
N. Wagatsuma, T. C. Potjans, M. Diesmann, K. Sakai, and T. Fukai. Spatial and feature-based attention in a layered cortical microcircuit model. [PLoS ONE](#), 8(12):e80788, 12 2013.

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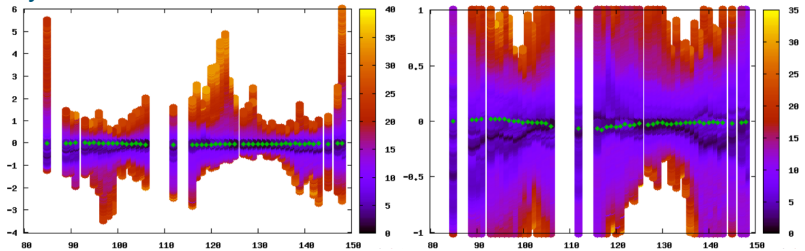
Tb^{3+}

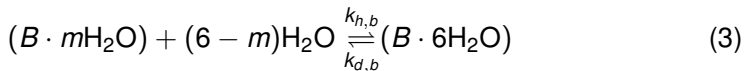
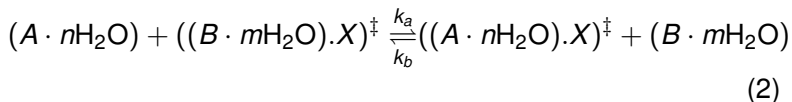
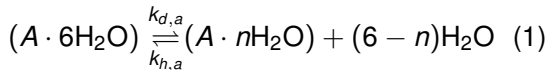


Tm^{3+}



Dy^{3+}





Charges in dielectric:

$$\rho^i(\mathbf{r}) = \frac{1 - \epsilon(\mathbf{r})}{\epsilon(\mathbf{r})} \rho^s(\mathbf{r}) - \frac{\nabla \epsilon(\mathbf{r})}{\epsilon(\mathbf{r})} \cdot \epsilon_0 \mathbf{E}(\mathbf{r})$$

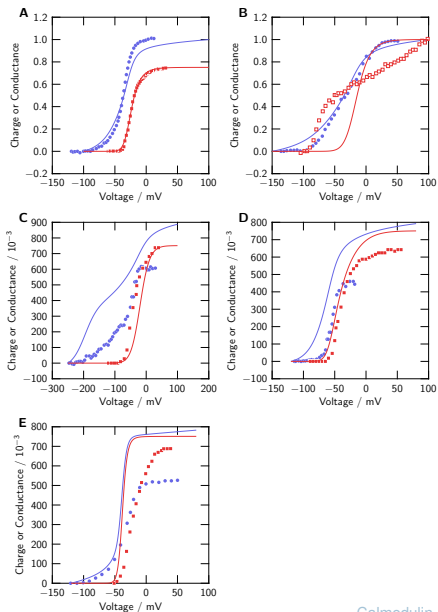
$$\rho^e(\mathbf{r}) = \frac{\rho^s(\mathbf{r})}{\epsilon(\mathbf{r})}.$$

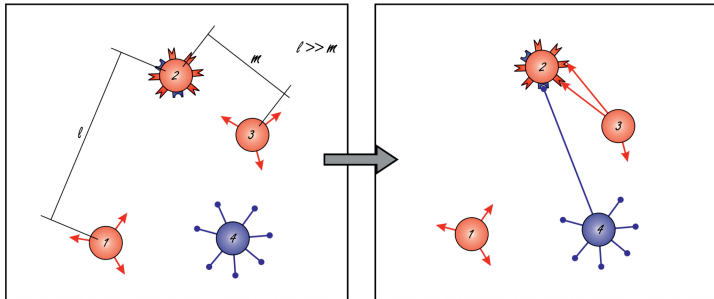
Matrix equations:

$$4\pi\epsilon_0 \mathbf{E}(\mathbf{r}) = \sum_k q_k^e \frac{\mathbf{r} - \mathbf{r}_k}{|\mathbf{r} - \mathbf{r}_k|^3} + \int_B \sigma^i(\mathbf{r}') \frac{\mathbf{r} - \mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|^3} d\mathbf{a}' + \int_E \sigma^e(\mathbf{r}') \frac{\mathbf{r} - \mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|^3} d\mathbf{a}'$$

$$4\pi\epsilon_0 V(\mathbf{r}) = \sum_k q_k^e \frac{1}{|\mathbf{r} - \mathbf{r}_k|} + \int_B \sigma^i(\mathbf{r}') \frac{1}{|\mathbf{r} - \mathbf{r}'|} d\mathbf{a}' + \int_E \sigma^e(\mathbf{r}') \frac{1}{|\mathbf{r} - \mathbf{r}'|} d\mathbf{a}'$$

$$\sigma^i(\mathbf{r}) = -\frac{\Delta \epsilon(\mathbf{r})}{\bar{\epsilon}(\mathbf{r})} \epsilon_0 \mathbf{n}(\mathbf{r}) \cdot \mathbf{E}(\mathbf{r})$$





Markus Butz-Ostendorf